Advanced Computational Analysis

ACA

REPORT

REPORT NO: S2149-2

Revision A

Title: Closed-Form Analysis Of Forces And Moments In Bungee Trampoline Structure

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Summary

The results of the closed-form calculations for the loads transferred to the system from the bungee ropes substantiate those predicted by the finite element results. Whilst the forces and bending moments predicted by this analysis for the aluminium poles are slightly different from those predicted by the finite element analysis, it is clear that the finite element model is significantly more accurate and provides a close approximation to the true system.

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Method Of Analysis

The analysis detailed below provides a closed-form estimate of the forces and moments which occur in a bungee trampoline structure, when a single passenger bounces vertically with a pre-defined vertical acceleration. This applied force is assumed to be a best estimate of the maximum force, since at the bottom of the descent the passenger is arrested by a trampoline.

The forces and moments derived in the analysis detailed below are based on a linear system of constraints and stiffness and a number of assumptions have to be made in order to provide an estimate of these forces and moments.

Assumptions

i) The structural system is linear and any frictional resistance is excluded.

ii) The motion of the passenger is purely vertical.

iii) Any restraint ropes offer no resistance to the deflexion of the structural frame.

iv) The self-weight of the structure is omitted.

v) The analysis assumes that only one passenger is active on the overall structure. Any additional load from other passengers bouncing simultaneously with the first passenger can be derived from kinematic summation of the forces in calculation sheets 1 to 4.

The forces and moments in the aluminium poles are verified in calculation sheets 1 to 7 below.

The results of the closed form calculations are compared with finite element results in table 1.0 below

Results

Item	Closed-Form Result	Finite Element Result
Vertical Reaction At Pulley R_V (N)	1705	1726
Horizontal Reaction At Pulley R_{HA} (N)	381	379
Horizontal Reaction At Pulley R_{BA} (N)	881	962
Vertical Force At Motor R_{VM} (N)	1645	1335
Horizontal Force At Motor R_{VM} (N)	763	757
Axial Force In Pole R_X (N)	1903	4710
Bending Moment In Pole About z-z axis M_{ZZMAX} (Nm)	3112	317
Bending Moment In Pole About y-y axis M_{YYMAX} (Nm)	1947	213

Table 1 – Summary Of Results For Stresses, Utilisation Factors Deflexions And Base Reaction Forces

Conclusions

The results of the closed-form calculations show that the resolved forces at the top of the aluminium poles are within 10 % of those predicted by the finite element analysis. In addition to this the resolved forces at the motor are within 20 % of each other. The small discrepancy in the results is due to the method used to model the pulleys at the top of the aluminium poles in the finite element analysis. The finite element analysis uses torsional spring elements with a low stiffness value, which enables the solution to converge. In using the torsional springs some of the tension in the bungee rope is removed from the bungee rope and transferred to the aluminium poles via a bending moment. However, since the results are within 20% of each other it is clear that the finite element model is a close approximation of the true situation.

Whilst the closed-form results for the resolved forces at the pulley and motor substantiate those predicted by the finite element analysis, the axial force and bending moments predicted in the aluminium poles are significantly different. The discrepancy is primarily due to the assumptions made above in that the system acts linearly and constraints are kinematically sufficient i.e. the effect of the cables has been neglected. In reality these cables would effectively act as a prop for the aluminium pole thereby significantly reducing the bending moments predicted by the closed-form calculations. In addition to this, since the cables are angled downwards the tension in the cables would increase the axial force in the aluminium poles, as predicted by the finite element analysis

In conclusion, the results of the closed-form calculations for the loads transferred to the system from the bungee ropes substantiate those predicted by the finite element results. Whilst the forces and bending moments predicted by this analysis for the aluminium poles are different from those predicted by the finite element analysis, it is clear that the finite element model is significantly more accurate and provides a close approximation to the true system.

M. Lacey

Dr M. Lacey

Figures

Figure 1.1 – Typical View Of Bungee Trampoline

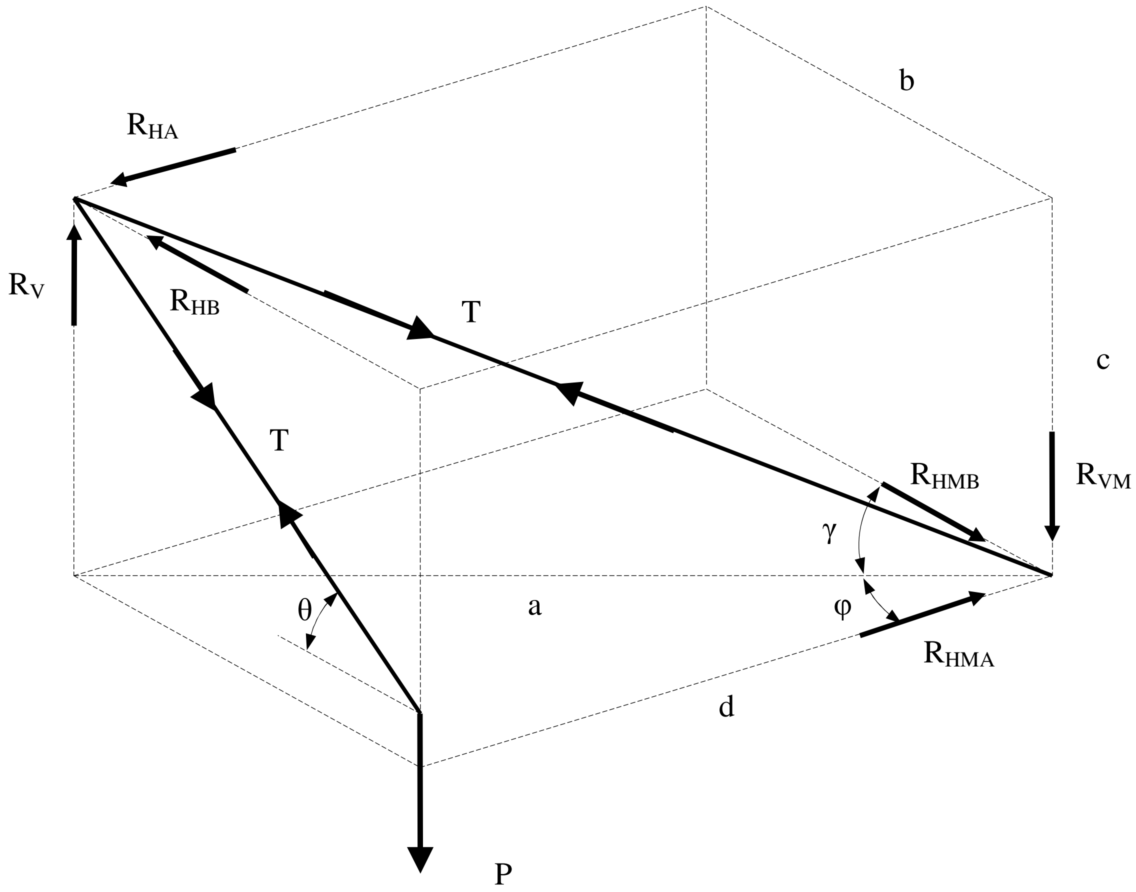

Figure 2.1 – Key To Location Of Symbols For Bungee Trampoline Arm

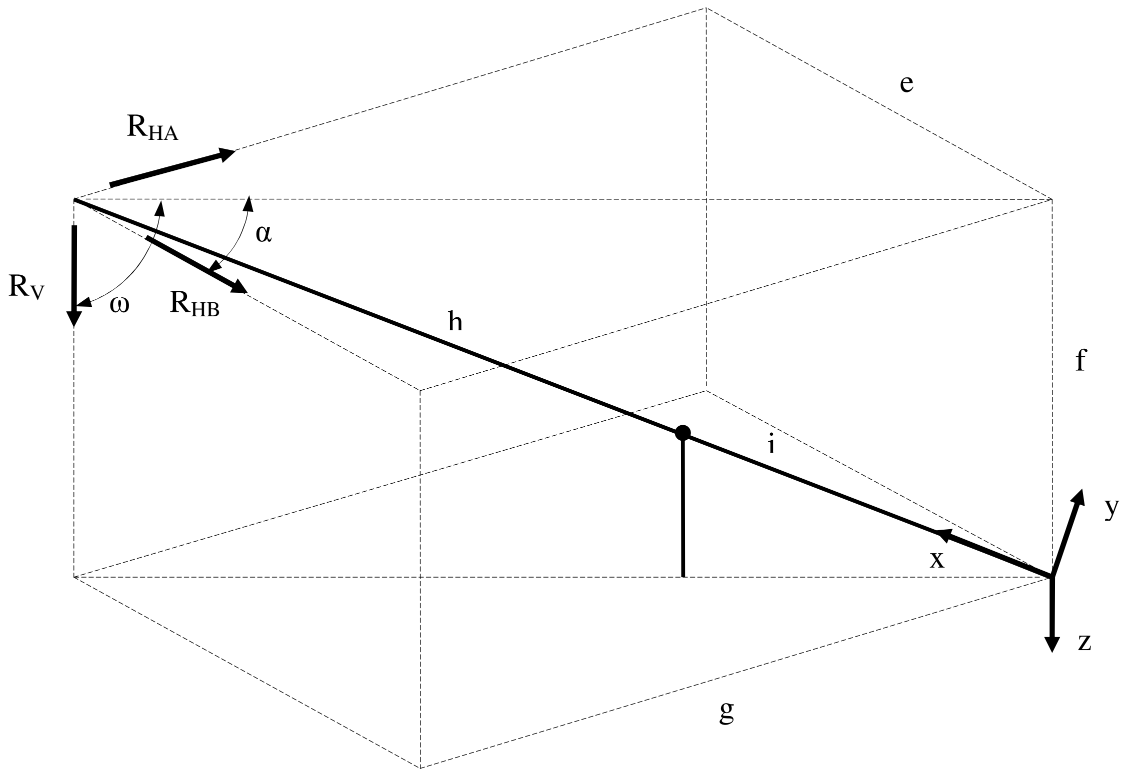

Figure 2.2 – Key To Location Of Symbols For Bungee Trampoline Arm

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Client :*Airmax Inflatable* ACA Contract No : *S2149-2*

Date : 27th February *2013*

Description : *Closed-Form Calculations For Trailer Mounted 4-Person Bungee Trampoline*

1.0 *Dimensions and Loads*

1.1 *1) Dimensions*

$a = 5.069\ m$ $b = 2.872\ m$ $c = 5.189\ m$ $d = 2.407\ m$

$e = 2.508\ m$ $f = 4.839\ m$ $g = 2.508\ m$ $h = 4.08\ m$

2) Loads

The combined mass of the passenger and harness is 90 kg. It is assumed that the maximum vertical acceleration of the passenger is 2 g. Hence the total vertical load P into one side of the system is

$$P = 90 \times 9.81 = 883\ N$$

Determination of reaction forces at pulley

1) Tension in bungee rope.

Referring to figure 2.1

tension in bungee rope is given by

$T sin(\theta) = P$ *P is half the total load from the passenger*

$$tan(\theta) = \frac{a}{b} \qquad sin(\theta) = \frac{a}{\sqrt{a^2 + b^2}} \qquad cos\theta = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\frac{T \times a}{\sqrt{a^2 + b^2}} = P$$

Therefore

$$T = \frac{P \times \sqrt{a^2 + b^2}}{a} = \frac{882.9 \times \sqrt{5.069^2 + 2.872^2}}{5.069} = 1014\ N$$

Prepared By: *R. Anderson* Checked By: *Dr M. Lacey*

 Section: 1 *Sheet: 1 of 7*

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2) Total vertical reaction at pulley

The total vertical reaction at pulley is given by

$$R_V = T \times sin(\theta) + T \times sin(\gamma)$$

$$tan(\gamma) = \frac{c}{\sqrt{b^2 + d^2}} \qquad sin(\gamma) = \frac{e}{\sqrt{b^2 + d^2 + c^2}}$$

$$cos(\gamma) = \frac{\sqrt{b^2 + d^2}}{\sqrt{b^2 + d^2 + c^2}}$$

$$R_V = T \times \frac{a}{\sqrt{a^2 + b^2}} + T \times \frac{c}{\sqrt{b^2 + d^2 + c^2}}$$

$$R_V = T\left(\frac{a}{\sqrt{a^2 + b^2}} + \frac{c}{\sqrt{b^2 + d^2 + c^2}}\right)$$

Therefore

$$R_V = P \times \left(1 + \frac{c}{a} \times \frac{\sqrt{a^2 + b^2}}{\sqrt{b^2 + d^2 + c^2}}\right) =$$

$$882.9 \times \left(1 + \frac{5.189}{5.069} \times \frac{\sqrt{5.069^2 + 2.872^2}}{\sqrt{2.872^2 + 2.407^2 + 5.189^2}}\right) = 1705\ N$$

The horizontal reaction R_{HA} at the pulley is given by

$$R_{HA} = T \times cos(\gamma) \times cos(\phi)$$

$$tan(\phi) = \frac{b}{d} \qquad sin(\phi) = \frac{b}{\sqrt{b^2 + d^2}} \qquad cos(\phi) = \frac{d}{\sqrt{b^2 + d^2}}$$

$$R_{HA} = \frac{T \times \sqrt{b^2 + d^2}}{\sqrt{b^2 + d^2 + c^2}} \times \frac{d}{\sqrt{b^2 + d^2}}$$

$$R_{HA} = \frac{T \times d}{\sqrt{b^2 + d^2 + c^2}}$$

Therefore

$$R_{HA} = P \times \frac{d}{a} \times \frac{\sqrt{a^2 + b^2}}{\sqrt{b^2 + d^2 + c^2}} = 882.9 \times \frac{2.407}{5.069} \times \frac{\sqrt{5.069^2 + 2.872^2}}{\sqrt{2.872^2 + 2.407^2 + 5.189^2}} = 381\ N$$

The horizontal reaction R_{HB} at the pulley is given by

$$R_{HB} = T \times cos(\gamma) \times sin(\phi) + T \times cos(\theta)$$

$$R_{HB} = T\left(\frac{\sqrt{b^2 + d^2}}{\sqrt{b^2 + d^2 + c^2}} \times \frac{d}{\sqrt{b^2 + d^2}} + \frac{b}{\sqrt{a^2 + b^2}}\right)$$

$$R_{HB} = T\left(\frac{d}{\sqrt{b^2 + d^2 + c^2}} + \frac{b}{\sqrt{a^2 + b^2}}\right) =$$

$$P \times \frac{\sqrt{a^2 + b^2}}{a} \times \left(\frac{d}{\sqrt{b^2 + d^2 + c^2}} + \frac{b}{\sqrt{a^2 + b^2}}\right)$$

Therefore

$$R_{HB} = P \times \left(\frac{d \times \sqrt{a^2 + b^2}}{a \times \sqrt{b^2 + d^2 + c^2}} + \frac{b}{a}\right) =$$

$$882.9 \times \left(\frac{2.407 \times \sqrt{5.069^2 + 2.872^2}}{5.069 \times \sqrt{2.872^2 + 2.407^2 + 5.189^2}} + \frac{2.872}{5.069}\right) = 881 \ N$$

The forces at the motor can be stated from equilibrium and the use of symmetry

$$R_{VM} = 2\left(R_{V.} - P\right)$$

Therefore

$$R_{VM} = 2P \times \frac{c}{a} \times \frac{\sqrt{a^2 + b^2}}{\sqrt{b^2 + d^2 + c^2}} =$$

$$2 \times 882.9 \times \frac{5.189}{5.069} \times \frac{\sqrt{5.069^2 + 2.872^2}}{\sqrt{2.872^2 + 2.407^2 + 5.189^2}} = 1645 \ N$$

$$R_{HMA} = 2R_{HA}$$

Therefore

$$R_{HMA} = 2 \times P \times \frac{d}{a} \times \frac{\sqrt{a^2 + b^2}}{\sqrt{b^2 + d^2 + c^2}} =$$

$$2 \times 882.9 \times \frac{2.407}{5.069} \times \frac{\sqrt{5.069^2 + 2.872^2}}{\sqrt{2.872^2 + 2.407^2 + 5.189^2}} = 763 \ N$$

$$R_{VMB} = 0$$

The pulley forces must now be resolved into the local co-ordinate system of the bungee pole, to determine the axial force, shear force and bending moment distribution in the pole. In figure 2.2 the x, y, z co-ordinate system is local co-ordinate system for the pole

1) Resolving forces into x-direction

$$R_X = \left(R_{HA} \times sin(\alpha) + R_{HB} \times cos(\alpha)\right) \times sin(\omega) + R_V \times cos(\omega)$$

$$tan(\alpha) = \frac{g}{e} \qquad sin(\alpha) = \frac{g}{\sqrt{g^2 + e^2}} \qquad cos(\alpha) = \frac{e}{\sqrt{g^2 + e^2}}$$

$$tan(\omega) = \frac{\sqrt{g^2 + e^2}}{f} \qquad sin(\omega) = \frac{\sqrt{g^2 + e^2}}{\sqrt{g^2 + e^2 + f^2}} \qquad cos(\omega) = \frac{f}{\sqrt{g^2 + e^2 + f^2}}$$

$$R_X = \frac{\left(R_{HA} \times g + R_{HB} \times e\right)}{\sqrt{g^2 + e^2}} \times \frac{\sqrt{g^2 + e^2}}{\sqrt{g^2 + e^2 + f^2}} + \frac{R_V \times f}{\sqrt{g^2 + e^2 + f^2}}$$

Therefore

$$R_X = \frac{R_{HA} \times g + R_{HB} \times e + R_V \times f}{\sqrt{g^2 + e^2 + f^2}} =$$

$$\frac{381.61 \times 2.508 + 881.845 \times 2.508 + 1705.574 \times 4.839}{\sqrt{2.508^2 + 2.508^2 + 4.839^2}} = 1903\ N$$

axial force in pole

2) Resolving forces in y direction

$$R_Y = \left(R_{HA} \times sin(\alpha) + R_{HB} \times cos(\alpha)\right) \times cos(\omega) - R_V \times sin(\omega)$$

$$R_Y = \frac{\left(R_{HA} \times g + R_{HB} \times e\right) \times f}{\sqrt{g^2 + e^2} \times \sqrt{g^2 + e^2 + f^2}} - \frac{R_V \times \sqrt{g^2 + e^2}}{\sqrt{g^2 + e^2 + f^2}}$$

Therefore

$$R_Y = \frac{\left(R_{HA} \times g + R_{HB} \times e\right) \times f - R_V \times \left(g^2 + e^2\right)}{\sqrt{g^2 + e^2} \times \sqrt{g^2 + e^2 + f^2}} =$$

$$\frac{(381.61 \times 2.508 + 881.845 \times 2.508) \times 4.839 - 1705.574 \times \left(2.508^2 + 2.508^2\right)}{\sqrt{2.508^2 + 2.508^2} \times \sqrt{2.508^2 + 2.508^2 + 4.839^2}} = -287\ N$$

y direction shear in pole

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4) Bending moment in pole

The bending moment in the pole about the z-z axis is given by

$$M_{zzmax} = R_Y \times h$$

Therefore

$$M_{zzmax} = \frac{h \times \left[R_{HA} \times g + R_{HB} \times f - R_V \times \left(g^2 + e^2\right)\right]}{\sqrt{g^2 + e^2} \times \sqrt{g^2 + e^2 + f^2}} =$$

$$\frac{4.08 \times \left[381.61 \times 2.508 + 881.845 \times 4.839 - 1705.574 \times \left(2.508^2 + 2.508^2\right)\right]}{\sqrt{2.508^2 + 2.508^2} \times \sqrt{2.508^2 + 2.508^2 + 4.839^2}} = -3112\ Nm$$

The bending moment in the pole about the y-y axis is given by.

$$M_{yymax} = R_Z \times h$$

Therefore

$$M_{yymax} = \frac{\left(R_{HA} \times e \times h - R_{HB} \times g\right) \times h}{\sqrt{g^2 + e^2}} =$$

$$\frac{(381.61 \times 2.508 \times 4.08 - 881.845 \times 2.508) \times 4.08}{\sqrt{2.508^2 + 2.508^2}} = 1947\ Nm$$